Scattering and Mode Conversion of Guided Modes by a Spherical Object in an Optical Fiber

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Abstract—The scattering and the mode conversion of the guided modes due to a spherical object in a step-index optical fiber is analyzed theoretically. The incident fiber mode is expanded in terms of the spherical vector wave functions, and the scattered fields are obtained by applying the boundary conditions on the surface of the object with the aid of these expansions. The expressions for the total scattered power and the mode conversion coefficients are given. As an example, the scattering and mode conversion caused by a spherical air bubble are evaluated numerically.

I. Introduction

The problem of scattering and mode conversion of guided modes in an optical waveguide due to irregularities or impurity objects is of great interest from both the theoretical and practical points of view [1]–[7]. This paper deals with the scattering and mode conversion of the guided modes in a step-index optical fiber by a spherical object whose diameter is of the order of the wavelength of the incident light.

To treat the problem theoretically, the incident fiber mode is expanded in terms of the spherical vector wave functions, which makes it possible to use the conventional analytical method of obtaining the scattered fields by applying the boundary conditions on the surface of the spherical object. Since the refractive index difference between core and cladding is very small for most of the practically used step-index fibers, the multiple scattering effect caused by the reflection of the primary scattered waves at the core-cladding interface is ignored for simplicity.

The expressions for the total scattered power and the mode conversion coefficients are given. As a typical example, the scattering and mode conversion caused by a spherical air bubble in the core region are evaluated numerically.

II. Expansion of the Guided Modes in Terms of the Spherical Vector Wave Functions

The electric and magnetic fields of the linearly polarized (LP) modes of the fiber whose transverse electric field is polarized linearly in the y direction can be written in the form [8]

$$\overset{(1)}{\mathbb{E}}{}^{(2)} = A\left[i_y J_\nu(h\rho) \substack{\cos \\ \sin} \nu\phi + i_z \frac{jh}{2\beta}\left\{ J_{\nu+1}(h\rho) \substack{\sin \\ -\cos} (\nu+1)\phi + J_{\nu-1}(h\rho) \substack{\sin \\ -\cos} (\nu-1)\phi \right\}\right] e^{-j\beta z} \quad (1)$$

$$\overset{(1)}{\mathbb{H}}{}^{(2)} = -A\sqrt{\frac{\epsilon_0}{\mu_0}}\left[x_i \frac{\beta^2}{k_0|\beta|} J_\nu(h\rho) \substack{\cos \\ \sin} \nu\phi + i_z \frac{jh}{2k_0} \cdot \left\{ J_{\nu+1}(h\rho) \substack{\cos \\ \sin} (\nu+1)\phi - J_{\nu-1}(h\rho) \substack{\cos \\ \sin} (\nu-1)\phi \right\}\right] e^{-j\beta z} \quad (2)$$

in the core region, whereas in the cladding region,

$$\overset{(1)}{\mathbb{E}}{}^{(2)} = A\frac{J_\nu(hR)}{H_\nu^{(1)}(j\gamma R)}\left[i_y H_\nu^{(1)}(j\gamma\rho) \substack{\cos \\ \sin} \nu\phi - i_z \frac{\gamma}{2\beta}\left\{ H_{\nu+1}^{(1)}(j\gamma\rho) \substack{\sin \\ -\cos} (\nu+1)\phi + H_{\nu-1}^{(1)}(j\gamma\rho) \substack{\sin \\ -\cos} (\nu-1)\phi \right\}\right] e^{-j\beta z} \quad (3)$$

$$\overset{(1)}{\mathbb{H}}{}^{(2)} = A\frac{1}{k_0}\sqrt{\frac{\epsilon_0}{\mu_0}}\frac{J_\nu(hR)}{H_\nu^{(1)}(j\gamma R)}\left[-i_x \frac{\beta^2}{|\beta|} H_\nu^{(1)}(j\gamma\rho) \substack{\cos \\ \sin} \nu\phi + i_z \frac{\gamma}{2}\left\{ H_{\nu+1}^{(1)}(j\gamma\rho) \substack{\cos \\ \sin} (\nu+1)\phi - H_{\nu-1}^{(1)}(j\gamma\rho) \substack{\cos \\ \sin} (\nu-1)\phi \right\}\right] e^{-j\beta z}. \quad (4)$$

In the foregoing equations, ρ, ϕ, and z are the circular-cylindrical coordinates with z axis being the center axis of the fiber. i_x, i_y, and i_z represent the unit vectors directed along the positive x, y, and z axes, respectively, $J_\nu(h\rho)$ and $H_\nu^{(1)}(j\gamma\rho)$ are Bessel function and Hankel function of the first kind, respectively, where ν is zero or a positive integer, $h=\sqrt{k_1^2-\beta^2}$, $\gamma=\sqrt{\beta^2-k_2^2}$, $k_1=\omega\sqrt{\epsilon_1\mu_0}=n_1k_0$, $k_2=\omega\sqrt{\epsilon_2\mu_0}=n_2k_0$, k_0 is the free-space wavenumber, and β is the propagation constant. $n_1=\sqrt{\epsilon_1/\epsilon_0}$ and $n_2=\sqrt{\epsilon_2/\epsilon_0}$ are the refractive-indices of the core and cladding, respectively. A is the mode amplitude and R is

Manuscript received February 26, 1979.
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the core radius. The superscripts (1) and (2) on the left-hand side of the equations correspond to the upper and lower functions of ϕ on the right-hand side of the equations, respectively.

First, the center of the spherical object is assumed to be located at an arbitrary point O_1 on the x axis apart from the origin O by a distance l_0 (as shown in Fig. 1) in which the z axis corresponds to the fiber axis. By using the addition theorem of Bessel functions, the mode fields given by (1), (2) and (3), (4) can be expressed in terms of the circular-cylindrical coordinates (ρ_1, ϕ_1, z_1) whose origin coincides with O_1 and z_1 axis is parallel to the z axis. The electric field (1) in the core region, for example, then becomes

$$\overset{(1)}{\mathbb{E}^{(2)}} = A\sum_{s=0}^{\infty}\left(1-\frac{\delta_{s0}}{2}\right)\left[\begin{Bmatrix}\left(J_{\nu-s}(hl_0)+(-1)^s J_{\nu+s}(hl_0)\right)\Psi_1^{(1)}(r_1,s)\\ \left(J_{\nu-s}(hl_0)-(-1)^s J_{\nu+s}(hl_0)\right)\Psi_1^{(2)}(r_1,s)\end{Bmatrix} + \frac{j}{\beta l_0}\begin{Bmatrix}\left((\nu-s)J_{\nu-s}(hl_0)-(-1)^s(\nu+s)J_{\nu+s}(hl_0)\right)\Psi_2^{(1)}(r_1,s)\\ -\left((\nu-s)J_{\nu-s}(hl_0)+(-1)^s(\nu+s)J_{\nu+s}(hl_0)\right)\Psi_2^{(2)}(r_1,s)\end{Bmatrix}\right] \tag{5}$$

where

$$\overset{(1)}{\Psi_1^{(2)}}(r_1,s) = i_y J_s(h\rho_1)e^{-j\beta z}\,{}^{\cos}_{\sin}\, s\phi_1$$
$$\overset{(1)}{\Psi_2^{(2)}}(r_1,s) = i_z J_s(h\rho_1)e^{-j\beta z}\,{}^{\sin}_{\cos}\, s\phi_1. \tag{6}$$

In the foregoing equations, r_1 denotes the vector distance from the origin O_1 of the spherical coordinates (r_1, θ_1, ϕ_1) as shown in Fig. 1, s is a positive integer including zero, and δ_{mn} indicates the Kronecker's delta.

Let us introduce now the spherical vector wave functions $\mathbb{M}^{(i)}_{{}^{e}_{0}mn}$ and $\mathbb{N}^{(i)}_{{}^{e}_{0}mn}$ defined as [9]

$$\mathbb{M}^{(i)}_{{}^{e}_{0}mn} = \mathrm{rot}\left\{ i_r r z_n^{(i)}(kr)P_n^m(\cos\theta)\,{}^{\cos}_{\sin}\, m\phi\right\}$$
$$\mathbb{N}^{(i)}_{{}^{e}_{0}mn} = \frac{1}{k}\mathrm{rot}\,\mathbb{M}^{(i)}_{{}^{e}_{0}mn} \tag{7}$$

where $z_n^{(i)}(kr)$ represents the spherical Bessel functions $j_n(kr)$, $n_n(kr)$, $h_n^{(1)}(kr)$, and $h_n^{(2)}(kr)$ for $i=1$, 2, 3, and 4, respectively, and $P_n^m(\cos\theta)$ denotes the associated Legendre function. As long as weakly guiding fiber modes [8] are considered, the electromagnetic fields of the modes given by (1), (2) and (3), (4) can be expressed in good accuracy in terms of the vector functions $\mathbb{M}$ and $\mathbb{N}$ in the form [11]

$$\mathbb{E} = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\sum_{\sigma=e,0}\left(-j\frac{\omega}{k}C_{\sigma mn}\mathbb{M}^{(i)}_{\sigma mn} + D_{\sigma mn}\mathbb{N}^{(i)}_{\sigma mn}\right) \tag{8}$$

$$\mathbb{H} = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\sum_{\sigma=e,0}\left(\frac{1}{\mu}C_{\sigma mn}\mathbb{N}^{(i)}_{\sigma mn} + j\frac{k}{\omega\mu}D_{\sigma mn}\mathbb{M}^{(i)}_{\sigma mn}\right) \tag{9}$$

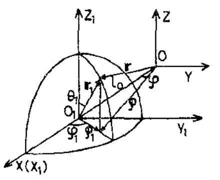

Fig. 1. System of coordinates used for the analysis.

where $C_{\sigma mn}$ and $D_{\sigma mn}$ are the expansion coefficients and the choice of i depends upon the type of field being considered. By using the cylindrical wave functions expressed in spherical coordinates [10] and the orthogonality properties of the function $\mathbb{M}$ and $\mathbb{N}$ [12], it can be shown that the expansion for the electric fields of the guided mode in the core region polarized linearly in the y direction is given by the following equation [13].

$$\overset{(1)}{\mathbb{E}^{(2)}} = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(-j\frac{\omega}{k_1}\overset{(1)}{C^{(2)}_{{}^{e}_{0}mn}}\mathbb{M}^{(1)}_{{}^{e}_{0}mn} + \overset{(1)}{D^{(2)}_{{}^{0}_{e}mn}}\mathbb{N}^{(1)}_{{}^{0}_{e}mn}\right) \tag{10}$$

where

$$-j\frac{\omega}{k_1}\overset{(1)}{C^{(2)}_{{}^{e}_{0}mn}} = -\Gamma_{mn}\overset{(1)}{W^{(2)}_{mn}},\ \overset{(1)}{D^{(2)}_{{}^{0}_{e}mn}} = j\Gamma_{mn}\left(\overset{(1)}{F^{(2)}_{mn}}\Lambda_1 + \overset{(1)}{G^{(2)}_{mn}}\Lambda_2\right) \tag{11}$$

and Γ_{mn}, $\overset{(1)}{W^{(2)}_{mn}}$, $\overset{(1)}{F^{(2)}_{mn}}$, $\overset{(1)}{G^{(2)}_{mn}}$, Λ_1, and Λ_2 are given in the Appendix.

Similar expression is also obtained for the electric field in the cladding region in which case k_1 is replaced by k_2, and $\overset{(1)}{P^{(2)}}(s)$ and $\overset{(1)}{Q^{(2)}}(s)$ are replaced by $\overset{(1)}{\hat{P}^{(2)}}(s)$ and $\overset{(1)}{\hat{Q}^{(2)}}(s)$, respectively, given by

$$\overset{(1)}{\hat{P}^{(2)}}(s) = A\frac{J_\nu(hR)}{H_\nu^{(1)}(j\gamma R)}\left\{H^{(1)}_{\nu-s}(j\gamma l_0) \pm (-1)^s H^{(1)}_{\nu+s}(j\gamma l_0)\right\}$$
$$\overset{(1)}{\hat{Q}^{(2)}}(s) = j\frac{A}{\beta l_0}\frac{J_\nu(hR)}{H_\nu^{(1)}(j\gamma R)}\left\{\pm(\nu-s)H^{(1)}_{\nu-s}(j\gamma l_0) - (-1)^s(\nu+s)H^{(1)}_{\nu+s}(j\gamma l_0)\right\}. \tag{12}$$

The corresponding expressions for the magnetic field in both the core and the cladding regions are automatically determined from (9).

Thus far, we have derived the expansion formula of the guided modes whose transverse electric field is polarized linearly in the y direction. A similar expansion formula of the guided modes whose transverse electric field lies in the x direction can be obtained by the same procedure [13]. It has been also assumed so far that the center of the spherical object, i.e. the origin O_1 of the spherical coordinates is on the x axis. However, we can treat the more general case where the center of the spherical object is located at an arbitrary position in the transverse (xy) plane by rotating the coordinate system and superposing both x and y polarized modes.

III. Scattered Wave from a Spherical Object

Let us consider the scattering of the incident fiber mode by a spherical object of radius a. It is assumed that the electric field of the incident guided mode is given by (10). Let the scattered and the transmitted fields $\mathbb{E}^s$, $\mathbb{H}^s$ and $\mathbb{E}^t$, $\mathbb{H}^t$, respectively, be expressed in the following form by means of the functions $\mathbb{M}$ and $\mathbb{N}$:

$$\mathbb{E}^{\overset{s}{t}} = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(-jc^{\overset{s}{t}}_{mn}\, \mathbb{M}^{\overset{(4)}{(1)}}_{emn} + d^{\overset{s}{t}}_{mn}\, \mathbb{N}^{\overset{(4)}{(1)}}_{0mn} \right) \quad (13)$$

$$\mathbb{H}^{\overset{s}{t}} = \frac{k_{\overset{1}{3}}}{\omega\mu_{\overset{1}{3}}} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(jd^{\overset{s}{t}}_{mn}\, \mathbb{M}^{\overset{(4)}{(1)}}_{0mn} + c^{\overset{s}{t}}_{mn}\, \mathbb{N}_{emn} \right)^{(4)}_{(1)} \quad (14)$$

where the subscripts 1 and 3 associated with k and μ in (14) correspond to the outer and inner regions of the spherical object, respectively. From the boundary conditions at $r_1 = a$

$$i_{r_1} \times (\mathbb{E}^{\text{inc}} + \mathbb{E}^s) = i_{r_1} \times \mathbb{E}^t, i_{r_1} \times (\mathbb{H}^{\text{inc}} + \mathbb{H}^s) = i_{r_1} \times \mathbb{H}^t \quad (15)$$

we obtain

$$c^s_{mn} = \Omega_1 c^{\text{inc}}_{mn}, d^s_{mn} = \Omega_2 d^{\text{inc}}_{mn} \quad (16)$$

where

$$\Omega_1 = -\frac{\mu_3 j_n(N\rho_0)[\rho_0 j_n(\rho_0)]' - \mu_1 j_n(\rho_0)[N\rho_0 j_n(N\rho_0)]'}{\mu_3 j_n(N\rho_0)[\rho_0 h_n^{(2)}(\rho_0)]' - \mu_1 h_n^{(2)}(\rho_0)[N\rho_0 j_n(N\rho_0)]'}$$

$$\Omega_2 = -\frac{\mu_3 j_n(\rho_0)[N\rho_0 j_n(N\rho_0)]' - \mu_1 N^2 j_n(N\rho_0)[\rho_0 j_n(\rho_0)]'}{\mu_3 h_n^{(2)}(\rho_0)[N\rho_0 j_n(N\rho_0)]' - \mu_1 N^2 j_n(N\rho_0)[\rho_0 h_n^{(2)}(\rho_0)]'}$$

$$c^{\text{inc}}_{mn} = \frac{\omega}{k_1} C^{(1)}_{emn}, d^{\text{inc}}_{mn} = D^{(1)}_{0mn}, \rho_0 = k_1 a, N = \frac{k_3}{k_1} \quad (17)$$

where the superscript "inc" stands for the incident mode fields and the prime notations refer to differentiation with respect to the total argument of the spherical function in the brackets. It should be noted that Ω_1 and Ω_2 given by (17) coincide with the well-known Mie coefficients. Only a small fraction of scattered wave (the wave which is scattered within the shaded region shown in Fig. 2) is reflected from the core-cladding boundary and illuminates the object again, giving rise to the secondary scattered wave and so on. However, these multiple scattering effects are ignored in the present analysis since the refractive index difference between core and cladding is remarkably small in an usual step-index fiber; furthermore, the solid angle of the shaded region shown in Fig. 2 is also usually very small.

The total scattered power is given by the following integral over an arbitrary spherical surface S_r of radius $r_1 > a$:

$$P^S = \tfrac{1}{2} R_e \int_{S_r} (\mathbb{E}^s \times \mathbb{H}^{s*}) \cdot i_{r_1} dS \quad (18)$$

which, by using the orthogonality relations $\mathbb{M}$ and $\mathbb{N}$ leads to

$$P^S = \frac{\pi}{2k_1\omega\mu_1} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(n+m)!}{(n-m)!} \frac{1}{g_n} \{ |c^s_{mn}|^2 + |d^s_{mn}|^2 \}. \quad (19)$$

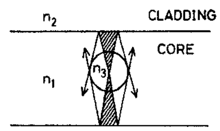

Fig. 2. Influence of higher order scattered waves.

IV. Mode Conversion Coefficients

Let us express both guided and radiation modes in the following simplified form:

$$\mathcal{E}_\mu = (e_{\mu t} + e_{\mu z}) e^{-j\beta_\mu z}, \mathcal{H}_\mu = (h_{\mu t} + h_{\mu z}) e^{-j\beta_\mu z} \quad (20)$$

where the subscripts t and z indicate the transverse and the longitudinal components, respectively, and the mode number (m,n) is represented simply by one term μ for brevity. For the radiation modes, $\mathcal{E}_\mu$ and $\mathcal{H}_\mu$ are functions of spectrum ζ as well as μ, but ζ is dropped for simplicity. The orthogonality relations which will be used in our analysis are [8]

$$\int_{S_t} (e_{\mu' t} \times h^*_{\mu t}) \cdot i_z \, dS = \begin{cases} 2P_\mu \dfrac{\beta_\mu}{|\beta_\mu|} \delta_{\mu'\mu}, & \text{for guided modes} \\ 2S_\mu(\zeta) P_\mu \dfrac{\beta^*_\mu}{|\beta_\mu|} \delta(\zeta - \zeta') \delta_{\mu'\mu}, & \text{for radiation modes} \end{cases} \quad (21)$$

where S_t represents an infinite transverse cross section plane. P_μ is the power carried by the μ mode, $\delta(\zeta)$ stands for the Dirac delta function, the asterisk indicates complex conjugation, and $S_\mu(\zeta)$ is $+1$ or -1 depending on whether β_μ is real or imaginary, respectively.

Suppose that the guided mode labeled μ' propagating in the positive z direction is scattered by a spherical object whose center is located at an arbitrary point O_1 on the transverse plane $z = 0$. Consider two infinite transverse cross sections S_1 and S_2 at $z = z_1 (<0)$ and $z = z_2 (>0)$, respectively, outside the spherical object as illustrated in Fig. 3. The scattered fields can be expanded on the planes S_1 and S_2 in terms of the modes as follows:

$$\mathbb{E}^s = \sum_\mu a^\pm_\mu \mathcal{E}^\pm_\mu + \sum_\mu \int R^\pm_\mu \mathcal{E}^\pm_\mu \, d\zeta$$

$$\mathbb{H}^s = \sum_\mu a^\pm_\mu \mathcal{H}^\pm_\mu + \sum_\mu \int R^\pm_\mu \mathcal{H}^\pm_\mu \, d\zeta \quad (22)$$

where the superscripts + and − represent the mode fields traveling in the positive and the negative z directions, respectively. We choose the volume v bounded by the surface S_0 of the spherical object, the two arbitrary cross sectional planes S_1 and S_2, and the cylindrical surface S_∞ at infinity. Then, from the Lorentz reciprocity principle in connection with the volume v, we get the following equation:

$$\int_{S_0 + S_1 + S_2} (\mathcal{E}_\mu \times \mathbb{H}^{s*} + \mathbb{E}^{s*} \times \mathcal{H}_\mu) \cdot i_n \, dS = 0 \quad (23)$$

owing to the fact that there are no sources in the region v

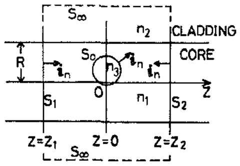

Fig. 3. Spherical surface S_0 and surfaces S_1, S_2, and S_∞ by which S_0 is enclosed.

and that the surface integral over S_∞ vanishes. If $\mathcal{E}_\mu$ and $\mathcal{H}_\mu$ in (23) are the fields of the mode traveling in the positive z direction (i.e. $\mathcal{E}_\mu^+$ and $\mathcal{H}_\mu^+$), the integrals over S_1 and S_2 can be carried out using the orthogonality relations of the modes, and hence a_μ^+ and R_μ^+ can be obtained from the integral over the remaining surface S_0 of the object. Similarly, if the mode traveling in the negative z direction ($\mathcal{E}_\mu^-$ and $\mathcal{H}_\mu^-$) is used, a_μ^- and R_μ^- can be obtained from the integral over S_0. Namely, provided that β_μ is real,

$$a_\mu^\pm, R_\mu^\pm = \frac{1}{4P_\mu} \oint_{S_0} \left(\mathcal{E}_\mu^{\pm *} \times \mathbb{H}^s + \mathbb{E}^s \times \mathcal{H}_\mu^{\pm *} \right) \cdot i_{r_1} dS \quad (24)$$

and

$$R_\mu^\pm = -\frac{j}{4P_\mu} \oint_{S_0} \left(\mathcal{E}_\mu^{\mp *} \times \mathbb{H}^s + \mathbb{E}^s \times \mathcal{H}_\mu^{\mp *} \right) \cdot i_{r_1} dS \quad (25)$$

provided that β_μ is imaginary. The upper or lower sign is to be taken throughout. By using the expansions for $\mathcal{E}_\mu$, $\mathcal{H}_\mu$ and $\mathbb{E}^s$, $\mathbb{H}^s$ in terms of $\mathbb{M}$ and $\mathbb{N}$ given, for example, by (10) and (13), together with the orthogonality relations of $\mathbb{M}$ and $\mathbb{N}$, the integrations in (24) and (25) can be carried out. If β_μ is real, for example, the result is

$$a_\mu^\pm = \frac{\pi}{r P_\mu \omega k_1 \mu_1} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(n+m)!}{(n-m)!} \frac{1}{g_n} \cdot \left(\Omega_1 c_{mn}^{\pm \mu} c_{mn}^{\mu'} + \Omega_2 d_{mn}^{\pm \mu} d_{mn}^{\mu\,\prime} \right). \quad (26)$$

V. Numerical Example

As an example of application, let us calculate numerically the scattering and the mode conversion of the fiber mode due to a spherical air bubble of radius a in a core region of the step-index fiber. Let

$$n_1 = 1.5, n_2 = (1 - 0.005) n_1, n_3 = 1, k_0 R = 133.5$$

and hence $V = \sqrt{n_1^2 - n_2^2}\, k_0 R = 20$. It is assumed that the incident mode is an LP_{01} mode whose electric field is given by $\mathcal{E}_{01}$. The power of all modes P_μ is defined as unity (1 W).

Fig. 4 shows the total scattered power calculated from (19) and the mode conversion coefficients given by (26) as a function of $k_0 a$ for the case of $l_0 = 0$. The mode conversion coefficients $|a_{mn}|^2$ between the incident LP_{01} mode and the coupled $LP_{mn}^\pm$ modes are indicated by the symbol LP_{mn}^+ or LP_{mn}^- in the Figure. The coupling between LP_{01} and LP_{11} modes does not occur in the case of $l_0 = 0$ and hence the curves corresponding to $LP_{11}^\pm$ does not appear in Fig. 4.

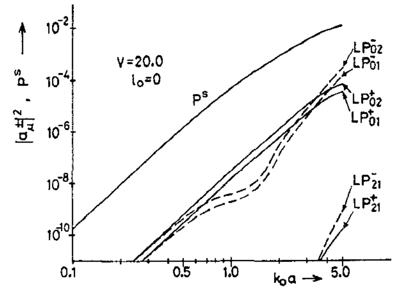

Fig. 4. Mode conversion coefficients and total scattered power $P^s \cdot (l_0 = 0)$.

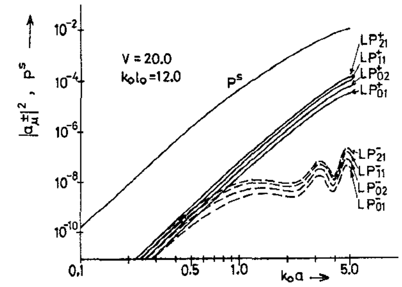

Fig. 5. Mode conversion coefficients and total scattered power $P^s \cdot (k_0 l_0 = 12.0)$.

TABLE I
Scattering Coefficient S_c: Comparison with Rayleigh Scattering Approximation

$k_0 a$	Rayleigh scattering	The value obtained from (19)
0.01	0.6973×10^{-8}	0.6972×10^{-8}
0.02	0.1116×10^{-6}	0.1115×10^{-6}
0.05	0.4358×10^{-5}	0.4340×10^{-5}
0.10	0.6973×10^{-4}	0.6855×10^{-4}
0.30	0.5648×10^{-2}	0.4876×10^{-2}
0.50	0.4358×10^{-1}	0.2990×10^{-1}

The case of $k_0 l_0 = 12.0$ is shown in Fig. 5 where all the numerical parameters used for the calculation are the same as those in the case of Fig. 4.

In our numerical example, the relative error of the scattered power caused by ignoring the effect of multiple scattering at the core-cladding interface is estimated to be less than the order of $0(10^{-6})$.

Table I shows a numerical comparison of the scattering coefficient $S_c (= \sigma_c / \pi a^2$ where σ_c is the total scattering cross section) obtained from (19) and that from the Rayleigh scattering approximation. As we can see from the Table, the total scattered power calculated from (19) is in agreement with that given by the Rayleigh scattering approximation if the radius of the spherical air bubble is small enough in comparison with the wavelength, i.e., $k_0 a \ll 1$.

VI. Conclusion

The guided mode of a step-index fiber has been expanded in terms of the spherical vector wave functions, and the scattering and the mode conversion effects caused by a spherical object have been analyzed theoretically by applying the conventional boundary conditions on the surface of the spherical object whose dimension is not necessarily small in comparison with a wavelength of the light. As an example, the scattering and the mode conversion due to a spherical air bubble have been calculated numerically.

The technique of mode expansion in terms of the spherical vector wave functions shown in this paper seems to be useful in analyzing a wide variety of similar scattering and mode conversion problems.

Appendix

$$\Gamma_{mn}=(-j)^{n-m+1}(n-m)!/(n+m)!$$

$$\Lambda_1=(n+1)j_{n-1}^2(k_1r_1)/\Lambda_0, \qquad \Lambda_2=nj_{n+1}^2(k_1r_1)/\Lambda_0$$

$$\Lambda_0=(n+1)j_{n-1}^2(k_1r_1)+nj_{n+1}^2(k_1r_1) \qquad \text{(A1)}$$

$$W_{mn}^{(1)}=\bar{U}_1(m-1)\cdot B_1+\bar{U}_1(m+1)\cdot B_2+\bar{U}_2(m)\cdot B_3$$

$$W_{mn}^{(2)}=\bar{V}_1(m-1)\cdot(1-\delta_{m1})B_1+(1-\delta_{m0})\bar{V}_1(m+1)\cdot B_2+\bar{V}_2(m)\cdot B_3 \qquad \text{(A2)}$$

$$B_1=P_n^{m-1}(\cos\alpha)\cdot(n-m+1)(n+m)$$

$$B_2=P_n^{m+1}(\cos\alpha), \qquad B_3=2mP_n^m(\cos\alpha) \qquad \text{(A3)}$$

$$F_{mn}^{(1)}=\bar{U}_1(m-1)\cdot D_1-\bar{U}_1(m+1)\cdot(1-\delta_{m0})D_2-2\bar{U}_2(m)\cdot(1-\delta_{m0})D_3$$

$$F_{mn}^{(2)}=-\bar{V}_1(m-1)\cdot(1-\delta_{m1})D_1+\bar{V}(m+1)\cdot D_2+\bar{V}_2(m)\cdot(2-\delta_{m0})D_3 \qquad \text{(A4)}$$

$$D_1=P_{n-1}^{m-1}(\cos\alpha)\cdot(n+m-1)(n+m)$$

$$D_2=P_{n-1}^{m+1}(\cos\alpha), \qquad D_3=P_{n-1}^m(\cos\alpha)\cdot(n+m) \qquad \text{(A5)}$$

$$G_{mn}^{(1)}=\bar{U}_1(m-1)\cdot T_1-\bar{U}_1(m+1)\cdot(1-\delta_{m0})T_2+2\bar{U}_2(m)\cdot(1-\delta_{m0})T_3$$

$$G_{mn}^{(2)}=-\bar{V}_1(m-1)\cdot(1-\delta_{m1})T_1+\bar{V}_1(m+1)\cdot T_2-\bar{V}_2(m)\cdot(2-\delta_{m0})T_3 \qquad \text{(A6)}$$

$$T_1=P_{n+1}^{m-1}(\cos\alpha)\cdot(n-m+2)(n-m+1)$$

$$T_2=P_{n+1}^{m+1}(\cos\alpha), \qquad T_3=P_{n+1}^m(\cos\alpha)\cdot(n-m+1) \qquad \text{(A7)}$$

$$\bar{U}_1(m)=g_nP^{(1)}(m), \qquad \bar{U}_2(m)=-jg_nQ^{(1)}(m),$$

$$\bar{V}_1(m)=g_nP^{(2)}(m), \qquad \bar{V}_2(m)=jg_nQ^{(2)}(m) \qquad \text{(A8)}$$

$$g_n=(2n+1)/\{2n(n+1)\}, \cos\alpha=\beta/k_1 \qquad \text{(A9)}$$

$$P^{\binom{(1)}{(2)}}(s)=A\{J_{\nu-s}(hl_0)\pm(-1)^sJ_{\nu+s}(hl_0)\}$$

$$Q^{\binom{(1)}{(2)}}(s)=\frac{jA}{\beta l_0}\{\pm(\nu-s)J_{\nu-s}(hl_0)-(-1)^s(\nu+s)J_{\nu+s}(hl_0)\}. \qquad \text{(A10)}$$

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